Problem 1

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 自反 | 对称 | 反对称 | 传递 |
| a | × | × | √ | √ |
| b | √ | √ | × | √ |
| c | √ | √ | × | √ |
| d | √ | √ | × | × |

Problem 2

“取元素b∈A使得{a, b}∈R”, 可能A中不存在这样的b.

Problem 3

a) M R1∪R2 = b) M R1∩R2 = c) M R2°R1 =

MR1∨MR2 = MR1∧MR2 = MR1⊙MR2 =

[0 1 0] [0 1 0] [0 1 1]

[1 1 1] [0 1 1] [1 1 1]

[1 1 1] [1 0 0] [0 1 0]

d) M R1°R2 = e) M R1⊕R2 =

MR1⊙MR1 = MR1⊕MR2 =

[1 1 1] [0 0 0]

[1 1 1] [1 0 0]

[0 1 0] [0 1 1]

Problem 4

a) W0 = W1 = W2 = W3 = W4 = W5 =

[0 0 1 0 0] [0 0 1 0 0] [0 0 1 0 0] [1 0 1 0 0] [1 0 1 0 0] [1 0 1 0 0]

[0 0 0 1 0] [0 0 0 1 0] [0 0 0 1 0] [0 0 0 1 0] [0 1 0 1 0] [0 1 0 1 0]

[1 0 0 0 0] [1 0 1 0 0] [1 0 1 0 0] [1 0 1 0 0] [1 0 1 0 0] [1 0 1 0 0]

[0 1 0 0 0] [0 1 0 0 0] [0 1 0 1 0] [0 1 0 1 0] [0 1 0 1 0] [0 1 0 1 0]

[0 0 0 1 0] [0 0 0 1 0] [0 0 0 1 0] [0 0 0 1 0] [0 1 0 1 0] [0 1 0 1 0]

{(a, a), (a, c), (b, b), (b, d), (c, a), (c, c), (d, b), (d, d), (e, b), (e, d)}

b) W0 = W1 = W2 = W3 = W4 = W5 =

[0 0 0 0 0] [0 0 0 0 0] [0 0 0 0 0] [0 0 0 0 0] [0 0 0 0 0] [0 0 0 0 0]

[0 0 1 0 1] [0 0 1 0 1] [0 0 1 0 1] [0 0 1 0 1] [0 0 1 0 1] [0 1 1 0 1]

[0 0 0 0 1] [0 0 0 0 1] [0 0 0 0 1] [0 0 1 0 1] [0 0 1 0 1] [0 1 1 0 1]

[1 0 0 0 0] [1 0 0 0 0] [1 0 0 0 0] [1 0 0 0 0] [1 0 0 0 0] [1 0 0 0 0]

[0 1 1 0 0] [0 1 1 0 0] [0 1 1 0 1] [0 1 1 0 1] [0 1 1 0 1] [0 1 1 0 1]

{(b, b), (b, c), (b, e), (c, b), (c, c), (c, e), (d, a), (e, b), (e, c), (e, e)}

c) W0 = W1 = W2 = W3 = W4 = W5 =

[0 1 1 0 1] [0 1 1 0 1] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 1 1]

[1 0 1 0 0] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 1 1]

[1 1 0 0 0] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 1 1]

[1 0 0 0 0] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 0 1] [1 1 1 1 1] [1 1 1 1 1]

[0 0 0 1 0] [0 0 0 1 0] [0 0 0 1 0] [0 0 0 1 0] [1 1 1 1 1] [1 1 1 1 1]

{(a, a), (a, b), (a, c), (a, d), (a, e), (b, a), (b, b), (b, c), (b, d), (b, e), (c, a), (c, b), (c, c), (c, d),

(c, e), (d, a), (d, b), (d, c) (d, d), (d, e), (e, a), (e, b), (e, c) (e, d), (e, e)}

d) W0 = W1 = W2 = W3 = W4 = W5 =

[0 0 0 0 1] [0 0 0 0 1] [0 0 0 0 1] [0 0 0 0 1] [0 0 0 0 1] [1 1 1 1 1]

[1 0 0 1 0] [1 0 0 1 1] [1 1 0 1 1] [1 0 0 1 1] [1 0 1 1 1] [1 1 1 1 1]

[0 0 0 1 0] [0 0 0 1 0] [0 0 0 1 0] [0 0 0 1 0] [1 0 1 1 1] [1 1 1 1 1]

[1 0 1 0 0] [1 0 1 0 1] [1 0 1 0 1] [1 0 1 1 1] [1 0 1 1 1] [1 1 1 1 1]

[1 1 1 0 1] [1 1 1 0 1] [1 1 1 1 1] [1 1 1 1 1] [1 1 1 1 1] [0 1 1 1 1]

{(a, a), (a, b), (a, c), (a, d), (a, e), (b, a), (b, b), (b, c), (b, d), (b, e), (c, a), (c, b), (c, c), (c, d),

(c, e), (d, a), (d, b), (d, c) (d, d), (d, e), (e, a), (e, b), (e, c) (e, d), (e, e)}

Problem 5

a+b = b+a, 则((a, b), (a, b))∈R, R是自反的.

((a, b), (c, d))∈R, a+d = b+c, 即c+b = d+a, ((c, d), (a, b))∈R, R是对称的.

((a, b), (c, d)), ((c, d), (e, f))∈R, a+d = b+c且c+f = d+e,

则a+d+c+f = b+c+d+e, a+f = b+e, ((a, b), (e, f))∈R, R是传递的, 则R是等价关系.

Problem 6

x与x的前三位相同, (x, x)∈R, R自反.

(x, y)∈R, x与y的前三位相同, y与x的前三位相同, (y, x)∈R, R对称.

(x, y), (y, z)∈R, x与y的前三位相同, y与z的前三位相同,

则x与z的前三位相同, (x, z)∈R, R传递, 则R是等价关系.

Problem 7

n元素集合上有2^(n^2)个关系, 其中自反的有2^(n^2-n)

a) 对称的有2^n×2^C(n, 2) = 2^(n + n(n-1)/2) = 2^(n(n+1)/2)

b) 反对称的有2^n×3^C(n, 2) = 2^n×3^(n(n-1)/2)

c) 非对称的有2^(n^2) – 2^(n(n+1)/2)

d) 反自反的有2^(n^2-n)

e) 自反的和对称的有2^C(n, 2) = 2^(n(n-1)/2)

f) 既不自反也不对称的有2^(n^2) – 2^(n^2-n) – 2^(n(n+1)/2) + 2^(n(n-1)/2)

= 2^n(2^n–2^(n-1)–2^(n+1 / 2)+2^(n-1 / 2) = 2^n×(2^(n-1)–2^(n-1 / 2))

Problem 8

a) 关系R的对称闭包为(R∪R^-1), 它的自反闭包为(R∪R^-1)∪IA

关系R的自反闭包为(R∪IA), 它的为对称闭包为(R∪IA)∪(R∪IA)^-1

= (R∪IA)∪(R^-1∪IA^-1) =(R∪IA)∪(R^-1∪IA) = (R∪R^-1)∪IA

b) 假设(a, b)∈s(t(R)), 则(a, b)∈t(R)或(b, a)∈t(R).

则R中存在一条从a到b或从b到a的路径(步数不限),

则s(R)中存在从a到b和从b到a的两条路径, (a, b)∈t(s(R)).

Problem 9

a) 否, 可能不传递, 如{(1, 1), (1, 2), (2, 1), (2, 2)}与{(1, 1), (1, 3), (3, 1), (3, 3)}

b) 是, x∈S(R1∩R2), x∈S1且x∈S2, (x, x)∈R1且(x, x)∈R2, (x, x)∈R1∩R2

(x, y)∈R1∩R2, (x, y)∈S1且(x, y)∈S2, (y, x)∈R1且(y, x)∈R2, (y, x)∈R1∩R2

(x, y), (y, z)∈R1∩R2, (x, y), (y, z)∈S1且(x, y), (y, z)∈S2,

(x, z)∈R1且(x, z)∈R2, (x, z)∈R1∩R2, R1∩R2自反, 对称且传递.

c) 否, 可能不自反, 如{(1, 1)}与{(1, 1), (1, 2), (2, 1), (2, 2)}

Problem 10

否, 如{1, 2, 3}上的关系{(1, 2), (3, 2)}, 传递闭包为{(1, 2), (3, 2)},

传递闭包的自反闭包为{(1, 1), (1, 2), (2, 2), (3, 2), (3, 3)},

传递闭包的自反闭包的对称闭包为{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)},

(1, 2), (2, 3)都属于这个闭包, 但(1, 3)不属于, 闭包不传递, 不是等价关系.